

# ROBUST AUXILIARY VECTOR FILTERING WITH CONSTRAINED CONSTANT MODULUS DESIGN FOR BEAMFORMING

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## ABSTRACT

This paper proposes an auxiliary vector filtering (AVF) algorithm based on a constrained constant modulus (CCM) design for robust adaptive beamforming. This scheme provides an efficient way to deal with filters with a large number of elements. The proposed beamformer decomposes the adaptive filter into a constrained (reference vector filters) and an unconstrained (auxiliary vector filters) components. The weight vector is iterated by subtracting the scaling auxiliary vector from the reference vector. The scalar factor and the auxiliary vector depend on each other and are jointly calculated according to the CCM criterion. The proposed robust AVF algorithm provides an iterative exchange of information between the scalar factor and the auxiliary vector and thus leads to a fast convergence and an improved steady-state performance over the existing techniques. Simulations are performed to show the performance and the robustness of the proposed scheme and algorithm in several scenarios.

**Index Terms**— Beamforming, antenna arrays, constrained constant modulus, auxiliary vector.

## 1. INTRODUCTION

Adaptive beamforming techniques are widely used in numerous applications such as radar, wireless communications, and sonar [1]–[2], to detect or improve the reception of a desired signal while suppressing interference at the output of a sensor array. Currently, the beamformers designed according to the constrained minimum variance (CMV) and the constrained constant modulus (CCM) criteria are among the most used criteria due to their simplicity and effectiveness. The CMV criterion aims to minimize the beamformer output power while maintaining the array response on the direction of the desired signal. The CCM criterion is a positive measure (Chapter 6 in [2]) of the deviation of the beamformer output from a constant modulus (CM) condition subject to a constraint on the array response of the desired signal. Compared with the CMV, the CCM-based beamformers exhibit superior performance in many severe scenarios (e.g., steering vector mismatch) since the positive measure provides more information for parameter estimation with constant modulus signals.

For the design of adaptive beamformers, numerous adaptive filtering algorithms have been developed using constrained optimization techniques [3], [5]. The stochastic gradient (SG) and recursive least squares (RLS) [3], [5] are popular methods with different tradeoffs between performance and complexity. A major drawback is that they require a large number of samples to reach the steady-state when the array size is large. In dynamic scenarios, filters with many elements usually provide a poor performance in tracking signals embedded in interference and noise. The multistage Wiener filter (MSWF) [8] provides a way out of this dilemma. The MSWF employs the minimum mean squared error (MMSE) criterion and its

extended versions with the CMV and the CCM criteria are reported in [9], [10]. Another cost-effective technique is the auxiliary vector filtering (AVF) [11] algorithm. In this scheme, an auxiliary vector is calculated by maximizing the cross correlation between the outputs of the reference vector filter and the previously auxiliary vector filters. The weight vector is obtained by subtracting the scaling auxiliary vector from the reference vector. In [12], the AVF algorithm iteratively generates a sequence of filters that converge to the CMV filter with a small number of samples. Its application in adaptive beamforming has been reported in [13].

Motivated by the fact that the CCM-based beamformers outperform the CMV ones for the CM signals, we propose an AVF algorithm based on the CCM design for robust adaptive beamforming. The beamformer structure decomposes the adaptive filter into a constrained (reference vector filters) and an unconstrained components (auxiliary vector filters). The constrained component is initialized with the array response of the desired signal to start the iteration and to ensure the CCM constraint, and the auxiliary vector in the unconstrained component can be iterated to meet the CM criterion. The weight vector is computed by means of suppressing the scaling unconstrained component from the constrained part. The main difference from the existing AVF technique is that, in the proposed CCM-based algorithm, the auxiliary vector and the scalar factor depend on each other and are jointly calculated according to the CM criterion (subject to different constraints). The proposed method provides an iterative exchange of information between the auxiliary vector and the scalar factor and also exploits the information about the CM signals, which leads to an improved performance. Simulations exhibit the robustness of the proposed method in typical scenarios including array mismatches.

The rest of this paper is organized as follows: we outline a system model and the problem statement in Section 2. The proposed scheme is introduced and the CCM-AVF algorithm is developed in Section 3. Simulation results are provided and discussed in Section 4, and conclusions are drawn in Section 5.

## 2. SYSTEM MODEL AND CCM BEAMFORMER DESIGN

Consider  $q$  narrowband signals that impinge on a uniform linear array (ULA) of  $m$  ( $m \geq q$ ) sensor elements. The sources are assumed to be in the far field with directions of arrival (DOAs)  $\theta_0, \dots, \theta_{q-1}$ . The  $i$ th received vector  $\mathbf{x}(i) \in \mathbb{C}^{m \times 1}$  can be modeled as

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \quad i = 1, \dots, N, \quad (1)$$

where  $\boldsymbol{\theta} = [\theta_0, \dots, \theta_{q-1}]^T \in \mathbb{R}^{q \times 1}$  is the signal DOAs,  $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_0), \dots, \mathbf{a}(\theta_{q-1})] \in \mathbb{C}^{m \times q}$  comprises the signal steering vectors  $\mathbf{a}(\theta_k) = [1, e^{-2\pi j \frac{d}{\lambda_c} \cos \theta_k}, \dots, e^{-2\pi j (m-1) \frac{d}{\lambda_c} \cos \theta_k}]^T \in \mathbb{C}^{m \times 1}$ , ( $k = 0, \dots, q-1$ ), where  $\lambda_c$  is the wavelength and  $d$  is the inter-element distance of the ULA ( $d = \lambda_c/2$  in general),  $\mathbf{s}(i) \in \mathbb{C}^{q \times 1}$  is the source data,  $\mathbf{n}(i) \in \mathbb{C}^{m \times 1}$  is assumed to be a zero-mean

spatially white Gaussian process,  $N$  is the number of snapshots, and  $(\cdot)^T$  stands for transpose. To avoid mathematical ambiguities, the steering vectors  $\mathbf{a}(\theta_k)$  are normalized and considered to be linearly independents. The output of the beamformer is

$$y(i) = \mathbf{w}^H(i)\mathbf{x}(i), \quad (2)$$

where  $\mathbf{w}(i) = [w_1(i), \dots, w_m(i)]^T \in \mathbb{C}^{m \times 1}$  is the complex weight vector of the beamformer, and  $(\cdot)^H$  stands for Hermitian transpose.

With the signals introduced in (1) and (2), we can present the CCM beamformer design by minimizing the following cost function

$$J_{\text{cm}}(\mathbf{w}(i)) = \mathbb{E}\{[|y(i)|^2 - \nu]^2\}, \quad \text{subject to } \mathbf{w}^H(i)\mathbf{a}(\theta_0) = 1, \quad (3)$$

where  $\theta_0$  is the direction of the signal of interest (SOI) and  $\mathbf{a}(\theta_0)$  denotes the corresponding steering vector. The cost function is the expected deviation of the squared modulus of the array output to a constant, say  $\nu = 1$ . The constraint is set to maintain the power of the SOI and to ensure the convexity of the cost function. The weight expression obtained from (3) is

$$\mathbf{w}(i+1) = \mathbf{R}^{-1}(i) \left\{ \mathbf{p}(i) - \frac{[\mathbf{p}^H(i)\mathbf{R}^{-1}(i)\mathbf{a}(\theta_0) - 1]\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}^{-1}(i)\mathbf{a}(\theta_0)} \right\}, \quad (4)$$

where  $\mathbf{R}(i) = \mathbb{E}[|y(i)|^2 \mathbf{x}(i)\mathbf{x}^H(i)] \in \mathbb{C}^{m \times m}$ ,  $\mathbf{p}(i) = \mathbb{E}[y^*(i)\mathbf{x}(i)] \in \mathbb{C}^{m \times 1}$ , and  $(\cdot)^*$  denotes complex conjugate. Note that (4) is a function of previous values of  $\mathbf{w}(i)$  (since  $y(i) = \mathbf{w}^H(i)\mathbf{x}(i)$ ) and thus must be initialized to start the iteration. We keep the time index  $i$  in  $\mathbf{R}(i)$  and  $\mathbf{p}(i)$  for the same reason. The calculation of the weight vector is costly due to the matrix inversion. The SG or RLS type algorithms can be employed to reduce the computational load but suffer from a poor performance when the dimension  $m$  is large.

### 3. PROPOSED CCM BEAMFORMER DESIGN AND AVF ALGORITHM

In this section, we introduce a CCM-based adaptive filtering structure for beamforming and develop an efficient CCM-AVF algorithm for robust adaptive beamforming.

#### 3.1. Proposed CCM Beamformer

We define the cost function for the beamformer design, which is

$$J_{\text{av}}(\mathbf{w}(i)) = \mathbb{E}\{[\mathbf{w}^H(i)\tilde{\mathbf{x}}(i) - \nu]^2\}, \quad (5)$$

where  $\tilde{\mathbf{x}}(i) = \mathbf{y}^*(i)\mathbf{x}(i)$  can be viewed as a new received vector to the beamformer and  $\nu = 1$  is set in accordance with (3).

To obtain the weight solution for the time index  $i$ , we start the iteration by initializing the weight vector

$$\mathbf{w}_0(i) = \mathbf{a}(\theta_0)/\|\mathbf{a}(\theta_0)\|^2. \quad (6)$$

Then, we subtract a scaling auxiliary vector (unconstrained component) that is orthogonal to  $\mathbf{a}(\theta_0)$  from  $\mathbf{w}_0(i)$  (constrained component) and obtain

$$\mathbf{w}_1(i) = \mathbf{w}_0(i) - \mu_1(i)\mathbf{g}_1(i), \quad (7)$$

where  $\mathbf{g}_1(i) \in \mathbb{C}^{m \times 1}$  with  $\mathbf{g}_1^H(i)\mathbf{a}(\theta_0) = 0$ , and  $\mu_1(i)$  is a scalar factor to control the weight of  $\mathbf{g}_1(i)$ . The auxiliary vector is supposed to capture the signal components in  $\tilde{\mathbf{x}}(i)$  that are not from the direction  $\theta_0$ . The aim of (7) is to suppress the disturbance of the unconstrained component while maintaining the contribution of the SOI. The cost function in (5) appears in unconstrained form since the constraint has been incorporated in the weight adaptation.

#### 3.2. Proposed CCM-AVF Algorithm

From (7), it is necessary to determine the auxiliary vector  $\mathbf{g}_1(i)$  and the scalar factor  $\mu_1(i)$  for the calculation of  $\mathbf{w}_1(i)$ . Assuming  $\mathbf{g}_1(i)$  is known,  $\mu_1(i)$  can be obtained by minimizing  $\mathbb{E}\{[\mathbf{w}_1(i)\tilde{\mathbf{x}}(i) - 1]^2\}$ . Substituting (7) into this cost function, computing the gradient with respect to  $\mu_1(i)$  and equating it to zero, we obtain

$$\mu_1(i) = \frac{\mathbf{g}_1^H(i)\tilde{\mathbf{R}}(i)\mathbf{w}_0(i) - \mathbf{g}_1^H(i)\tilde{\mathbf{p}}(i)}{\mathbf{g}_1^H(i)\tilde{\mathbf{R}}(i)\mathbf{g}_1(i)}, \quad (8)$$

where  $\tilde{\mathbf{R}}(i) = \mathbb{E}[\tilde{\mathbf{x}}(i)\tilde{\mathbf{x}}^H(i)] \in \mathbb{C}^{m \times m}$  and  $\tilde{\mathbf{p}}(i) = \mathbb{E}[\tilde{\mathbf{x}}(i)] \in \mathbb{C}^{m \times 1}$ . Note that  $\mu_1(i) = 0$ , i.e.,  $\tilde{\mathbf{R}}(i)\mathbf{w}_0(i) = \tilde{\mathbf{p}}(i)$  needs to be avoided here since the design is equivalent to a matched filter if it happens.

On the other hand, the calculation of the auxiliary vector  $\mathbf{g}_1(i)$  should take the conditions  $\mathbf{g}_1^H(i)\mathbf{a}(\theta_0) = 0$  and  $\mathbf{g}_1^H(i)\mathbf{g}_1(i) = 1$  into account. The constrained minimization problem with respect to  $\mathbf{g}_1(i)$  can be transformed by the method of Lagrange multipliers into

$$J_L(\mathbf{w}_1(i)) = \mathbb{E}\{[\mathbf{w}_1^H(i)\tilde{\mathbf{x}}(i) - 1]^2\} - 2\Re\{\lambda_1[\mathbf{g}_1^H(i)\mathbf{g}_1(i) - 1] - \lambda_2\mathbf{g}_1^H(i)\mathbf{a}(\theta_0)\}, \quad (9)$$

where  $\lambda_1$  and  $\lambda_2$  are scalar Lagrange multipliers. For the sake of mathematical accuracy, we note that the cost function to be minimized is phase invariant, namely, if  $\mathbf{g}_1(i)$  satisfies it, so does  $\mathbf{g}_1(i)e^{j\phi}$  for any phase  $\phi$ . To avoid any ambiguity, we assume that only one auxiliary vector can be obtained.

Following the procedure to get  $\mu_1(i)$ , the auxiliary vector can be expressed by

$$\mathbf{g}_1(i) = \frac{\mu_1^*(i)\tilde{\mathbf{p}}_y(i) - \lambda_2\mathbf{a}(\theta_0)}{\lambda_1}, \quad (10)$$

where  $\tilde{\mathbf{p}}_y(i) = \mathbb{E}[(1 - \tilde{y}(i))^* \tilde{\mathbf{x}}(i)] \in \mathbb{C}^{m \times 1}$  and  $\tilde{y}(i) = \mathbf{w}^H(i)\tilde{\mathbf{x}}(i)$ . We keep the time index  $i$  in  $\tilde{\mathbf{p}}_y(i)$  since it is a function of  $\mathbf{w}(i)$ , which must be initialized to provide an estimation about  $\tilde{y}(i)$  and to start the iteration.

The expression of  $\mathbf{g}_1(i)$  is utilized to enforce the constraints and solve for  $\lambda_1$  and  $\lambda_2$ . Indeed, we have

$$\lambda_1 = \left\| \mu_1^*(i)\tilde{\mathbf{p}}_y(i) - \frac{\mu_1^*(i)\mathbf{a}^H(\theta_0)\tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2}\mathbf{a}(\theta_0) \right\|, \quad (11)$$

$$\lambda_2 = \frac{\mu_1^*(i)\mathbf{a}^H(\theta_0)\tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2}, \quad (12)$$

where  $\|\cdot\|$  denotes the Euclidean norm. Substitution of  $\lambda_1$  and  $\lambda_2$  back in (10) leads to  $\mathbf{g}_1(i)$  that satisfies the constraints and minimizes (with  $\mu_1(i)$ ) the squared deviation of  $\tilde{y}(i)$  from the CM condition, yielding

$$\mathbf{g}_1(i) = \frac{\mu_1^*(i)\tilde{\mathbf{p}}_y(i) - \frac{\mu_1^*(i)\mathbf{a}^H(\theta_0)\tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2}\mathbf{a}(\theta_0)}{\left\| \mu_1^*(i)\tilde{\mathbf{p}}_y(i) - \frac{\mu_1^*(i)\mathbf{a}^H(\theta_0)\tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2}\mathbf{a}(\theta_0) \right\|}. \quad (13)$$

So far, we have detailed the first iteration of the proposed CCM-AVF algorithm for time index  $i$ , i.e.,  $\mathbf{w}_0(i)$  in (6),  $\mathbf{w}_1(i)$  in (7),  $\mu_1(i)$  in (8), and  $\mathbf{g}_1(i)$  in (13), respectively. In this procedure,  $\tilde{\mathbf{x}}(i)$  can be viewed as a new received vector that is processed by the adaptive filter  $\mathbf{w}_1(i)$  (first estimation of  $\mathbf{w}(i)$ ) to generate the output  $\tilde{y}(i)$ , in which,  $\mathbf{w}_1(i)$  is determined by minimizing the mean squared error

**Table 1. PROPOSED CCM-AVF ALGORITHM**


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**For the time index**  $i = 1, 2, \dots, N$ .

**Initialization:**

$$\mathbf{w}(i) = \mathbf{w}_0(i) = \frac{\mathbf{a}(\theta_0)}{\|\mathbf{a}(\theta_0)\|^2}; \quad \mu_0(i) = \text{small positive value.}$$

**Iterative procedures:**

For  $k = 1, 2, \dots, K$

$$\mathbf{g}_k(i) = \mu_{k-1}^*(i) \tilde{\mathbf{p}}_y(i) - \frac{\mu_{k-1}^*(i) \mathbf{a}^H(\theta_0) \tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2} \mathbf{a}(\theta_0)$$

**if**  $\mathbf{g}_k(i) = \mathbf{0}$  **then EXIT.**

$$\mu_k(i) = \frac{\mathbf{g}_k^H(i) \tilde{\mathbf{R}}(i) \mathbf{w}_{k-1}(i) - \mathbf{g}_k^H(i) \tilde{\mathbf{p}}(i)}{\mathbf{g}_k^H(i) \tilde{\mathbf{R}}(i) \mathbf{g}_k(i)}$$

$$\mathbf{w}_k(i) = \mathbf{w}_{k-1}(i) - \mu_k \mathbf{g}_k(i)$$

**Weight expression:**

$$\mathbf{w}(i) = \mathbf{w}_K(i).$$


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between the output and the desired CM condition. This principle is suitable to the following iterations with  $\mathbf{w}_2(i)$ ,  $\mathbf{w}_3(i)$ ,  $\dots$

Now, we consider the iterations one step further and express the adaptive filter as

$$\mathbf{w}_2(i) = \mathbf{w}_0(i) - \sum_{k=1}^2 \mu_k(i) \mathbf{g}_k(i) = \mathbf{w}_1(i) - \mu_2(i) \mathbf{g}_2(i), \quad (14)$$

where  $\mu_2(i)$  and  $\mathbf{g}_2(i)$  will be calculated based on the previously identified  $\mu_1(i)$  and  $\mathbf{g}_1(i)$ .  $\mu_2(i)$  ( $\mu_2(i) \neq 0$ ) is chosen to minimize the cost function  $\mathbb{E}\{\|\mathbf{w}_2^H(i) \tilde{\mathbf{x}}(i) - 1\|^2\}$  under the assumption that  $\mathbf{g}_2(i)$  is known beforehand. Thus, we have

$$\mu_2(i) = \frac{\mathbf{g}_2^H(i) \tilde{\mathbf{R}}(i) \mathbf{w}_1(i) - \mathbf{g}_2^H(i) \tilde{\mathbf{p}}(i)}{\mathbf{g}_2^H(i) \tilde{\mathbf{R}}(i) \mathbf{g}_2(i)}, \quad (15)$$

The auxiliary vector  $\mathbf{g}_2(i)$  is calculated by the minimization of the cost function subject to the constraints  $\mathbf{g}_2^H(i) \mathbf{a}(\theta_0) = 0$  and  $\mathbf{g}_2^H(i) \mathbf{g}_2(i) = 1$ , which is

$$\mathbf{g}_2(i) = \frac{\mu_2^*(i) \tilde{\mathbf{p}}_y(i) - \frac{\mu_2^*(i) \mathbf{a}^H(\theta_0) \tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2} \mathbf{a}(\theta_0)}{\left\| \mu_2^*(i) \tilde{\mathbf{p}}_y(i) - \frac{\mu_2^*(i) \mathbf{a}^H(\theta_0) \tilde{\mathbf{p}}_y(i)}{\|\mathbf{a}(\theta_0)\|^2} \mathbf{a}(\theta_0) \right\|}. \quad (16)$$

The above iterative procedures are taken place at time index  $i$  to generate a sequence of filters  $\mathbf{w}_k(i)$  with  $k = 0, 1, \dots$  being the iteration number. Generally, there exists a maximum (or suitable) value of  $k$ , i.e.,  $k_{\max} = K$ , that is determined by a certain rule to stop iterations and achieve satisfactory performance. One simple rule, which is adopted in the proposed CCM-AVF algorithm, is to terminate the iteration if  $\mathbf{g}_k(i) \cong \mathbf{0}$  is achieved. Alternative and more complicated selection rules can be found in [13]. Until now, the weight solution at time index  $i$  can be given by  $\mathbf{w}(i) = \mathbf{w}_K(i)$ . The proposed CCM-AVF algorithm for the design of the CCM beamformer is summarized in Table 1.

### 3.3. Interpretations about Proposed CCM-AVF Algorithm

There are several points we need to interpret in Table 1. First of all, initialization is important to the realization of the proposed method.  $\mathbf{w}(i)$  is set to estimate  $\tilde{y}(i)$  and so  $\tilde{\mathbf{R}}(i)$ ,  $\tilde{\mathbf{p}}(i)$ , and  $\tilde{\mathbf{p}}_y(i)$ .  $\mathbf{w}_0(i)$  is for the activation of the weight adaptation. Note that the calculation of the scalar factor, e.g., in (8), is a function of  $\mathbf{g}_1(i)$  and the auxiliary vector obtained from (13) depends on  $\mu_1(i)$ . It is necessary to

initialize one of these quantities to start the iteration. We usually set a small positive scalar value  $\mu_0(i)$  for simplicity. Under this condition, the subscript of the scalar factor for the calculation of  $\mathbf{g}_k(i)$  should be replaced by  $k - 1$  instead of  $k$ , as shown in Table 1.

Second, the expected quantities  $\tilde{\mathbf{R}}(i)$ ,  $\tilde{\mathbf{p}}(i)$ , and  $\tilde{\mathbf{p}}_y(i)$  are not available in practice. We use a sample-average approach to estimate them, i.e.,

$$\begin{aligned} \hat{\tilde{\mathbf{R}}}(i) &= \frac{1}{i} \sum_{l=1}^i \tilde{\mathbf{x}}(l) \tilde{\mathbf{x}}^H(l); & \hat{\tilde{\mathbf{p}}}(i) &= \frac{1}{i} \sum_{l=1}^i \tilde{\mathbf{x}}(l); \\ \hat{\tilde{\mathbf{p}}}_y(i) &= \frac{1}{i} \sum_{l=1}^i (1 - \tilde{y}(l))^* \tilde{\mathbf{x}}(l). \end{aligned} \quad (17)$$

where  $\tilde{\mathbf{R}}(i)$ ,  $\tilde{\mathbf{p}}(i)$ , and  $\tilde{\mathbf{p}}_y(i)$  are substituted by their estimates in the iterative procedure to generate  $\mathbf{w}_k(i)$ . To improve the estimation accuracy, the quantities in (17) can be refreshed or further regularized during the iterations. Specifically, we use  $\mathbf{w}_k(i)$  in the iteration step instead of  $\mathbf{w}(i)$  in the initialization to generate  $y(i)$ , and related  $\tilde{\mathbf{x}}(i)$  and  $\tilde{y}(i)$ , which are employed to update the estimates  $\hat{\tilde{\mathbf{R}}}(i)$ ,  $\hat{\tilde{\mathbf{p}}}(i)$ , and  $\hat{\tilde{\mathbf{p}}}_y(i)$ . Compared with  $\mathbf{w}(i) = \mathbf{a}(\theta_0)/\|\mathbf{a}(\theta_0)\|^2$ ,  $\mathbf{w}_k(i)$  is more efficient to evaluate the desired signal. Thus, the refreshment of the estimates based on the current  $\mathbf{w}_k(i)$  is valuable to calculate the subsequent scalar factor and the auxiliary vector.

Third, we drop the normalization of the auxiliary vector [12]-[14]. Note that the calculated auxiliary vectors  $\mathbf{g}_k(i)$  are constrained to be orthogonal to  $\mathbf{a}(\theta_0)$ . The orthogonality among the auxiliary vectors is not imposed. Actually, the successive auxiliary vectors do satisfy the orthogonality as verifies in our numerical results. We omit the analysis about this characteristic here considering the paper length.

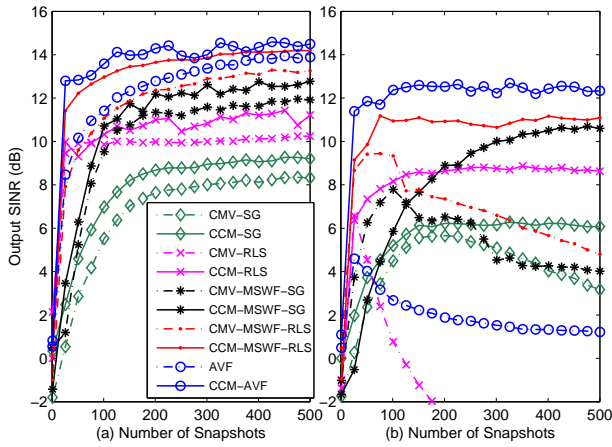
The proposed CCM-AVF beamformer efficiently measures the expected deviation of the beamformer output from the CM condition and provide useful information for the proposed algorithm for dealing with parameter estimation in many severe scenarios including low signal-to-noise ratio (SNR) or steering vector mismatch. The proposed CCM-AVF algorithm employs an iterative procedure to adjust the weight vector for each time instant. The matrix inversion appeared in (4) is avoided and thus the computational cost is limited. Since the scalar factor and the auxiliary vector depend on each other, the proposed algorithm provides an iterative exchange of information between them, which are jointly employed to update the weight vector. This scheme leads to an improved convergence and the steady-state performance that will be shown in the simulations.

## 4. SIMULATIONS

Simulations are performed for a ULA containing  $m = 40$  sensor elements with half-wavelength interelement spacing. We compare the proposed algorithm (CCM-AVF) with the SG [3], RLS [5], MSWF [10], and AVF [12] methods. With respect to each method, we consider the CMV and the CCM criteria for beamforming. A total of 1000 runs are used to get the curves. In all experiments, BPSK sources' powers (desired user and interferers) are  $\sigma_s^2 = \sigma_I^2 = 1$  and the input SNR = 0 dB with spatially and temporally white Gaussian noise.

Fig. 1 includes two experiments. There are  $q = 10$  users, including one desired user in the system. The scalar factor is  $\mu_0(i) = 0.01$  and the iteration number is  $K = 3$ . In Fig.1 (a), the exact DOA of the SOI is known at the receiver. All output SINR values

increase to the steady-state as the increase of the snapshots (time index). The RLS-type algorithms enjoy faster convergence and better steady-state performance than the SG-type methods. The proposed CCM-AVF algorithm converges rapidly and reaches the steady-state with superior performance. The CCM-based MSWF technique with the RLS implementation has comparative fast convergence rate but the steady-state performance is not better than the proposed method. In Fig. 1 (b), we set the DOA of the SOI estimated by the receiver to be  $1^\circ$  away from the actual direction. It indicates that the mismatch induces performance degradation to all the analyzed algorithms. The CCM-based methods are more robust to this scenario than the CMV-based ones. The proposed CCM-AVF algorithm has faster convergence and better steady-state performance than the other analyzed methods.

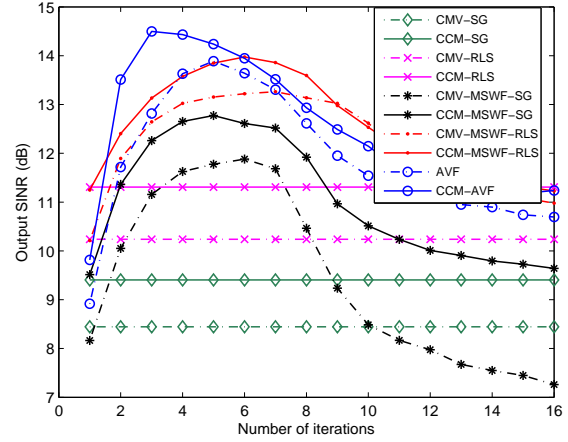


**Fig. 1.** Output SINR versus the number of snapshots for (a) ideal steering vector; (b) steering vector mismatch  $1^\circ$ .

In Fig. 2, we keep the same scenario as that in Fig. 1 (a) and check the iteration number for the existing and proposed methods. The number of snapshots is fixed to  $N = 500$ . The most adequate iteration number for the proposed CCM-AVF algorithm is  $K = 3$ , which is comparatively lower than other analyzed algorithms, but reach the preferable performance. We also checked that this value is rather insensitive to the number of users in the system, to the number of sensor elements, and work efficiently for the studied scenarios.

## 5. CONCLUDING REMARKS

We developed an AVF algorithm based on the CCM design for robust adaptive beamforming. The algorithm provides a positive measure of the expected deviation of the beamformer output from the CM condition and thus is robust against the severe scenarios. The weight solution is iterated by jointly calculating the auxiliary vector and the scalar factor, which iteratively exchange information between each other and lead to an improved performance over prior art. The selection of the iteration number may be more efficient and adaptive with the change of the system (e.g., the number of users change) if we employ other techniques [13]. We will consider further improvements to the proposed CCM-AVF algorithm in the near future.



**Fig. 2.** Output SINR versus the number of iterations.

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